Period 1, Nov 20, 2024

1) $y = \sin 3x$ $\quad \frac{dy}{dx} = (\cos 3x)\cdot(3)$

2) $y = (x^2 - 5x)^3$ $\quad \frac{dy}{dx} = 3(x^2-5x)^2(2x-5)$

3) $y = \sqrt{(7-6x^3)^5}$ $\quad \frac{dy}{dx} = \frac{5}{2}(7-6x^3)^{3/2}(-18x^2)$

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1. $y = \sin 3x \Rightarrow y = \sin u$

$u = 3x$

$\frac{du}{dx} = 3$

$\frac{dy}{du} = \cos u$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dx}$

$3(\cos u) = \frac{dy}{dx}$

$3\cos 3x = \frac{dy}{dx}$

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2. $y = (x^2-5x)^3 \Rightarrow y = u^3$

$u = x^2 - 5x$

$\frac{du}{dx} = 2x - 5$

$\frac{dy}{du} = 3u^2$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dx}$

$(2x-5)(3u^2) = \frac{dy}{dx}$

$(2x-5)(3)(x^2-5x)^2 = \frac{dy}{dx}$

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3. $y = \sqrt{(7-6x^3)^5} = (7-6x^3)^{5/2} \Rightarrow y = u^{5/2}$

$u = 7 - 6x^3$

$\frac{du}{dx} = -18x^2$

$\frac{dy}{du} = \frac{5}{2}u^{3/2}$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dx}$

$(-18x^2)\cdot\frac{5}{2}u^{3/2} = \frac{dy}{dx}$

$(-18x^2)\cdot\frac{5}{2}\cdot(7-6x^3)^{3/2}$

Related Rates

Take der with Respect To Time = T

$a^2 + b^2 = c^2$

$2a\frac{da}{dT} + 2b\frac{db}{dT} = 2c\frac{dc}{dT}$

"with Respect To"

b) $A = \pi r^2$ Change in Radius

$\frac{dA}{dT} = 2\pi r\frac{dr}{dT}$

with Respect To"

Change in Area with Respect To Time

c. $Tan\,\theta = \frac{x}{5}$

$(Sec^2\theta)\frac{d\theta}{dT} = \frac{1}{5}\frac{dx}{dT}$

d. $2y^2 - 3x^4 = 0$

$4y\frac{dy}{dT} - 12x^3\frac{dx}{dT} = 0$

Steps for Solving a Related Rate Problem

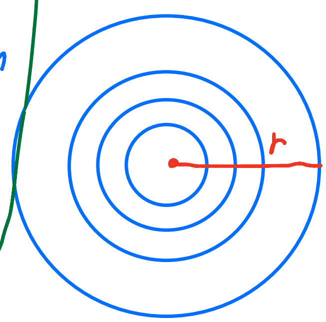
1. Draw a diagram
2. Write the rates you know
3. Write the rates you want
4. Get an equation related to the variables that appear in the rates
5. Differentiate (using the chain rule) and solve for the wanted rate.
6. Answer the question (with units)

Example 5: A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius *r* of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area *A* of the disturbed water changing?

4 feet $= r$

$A = \pi r^2$

$\frac{dA}{dT}$ = change in Area over Time



$\frac{dr}{dt}$ = change in Radius with Respect to Time

$\frac{dr}{dT} = 1$ Foot/second

$\frac{dr}{dt} = 1 \text{ ft/sec}$

$A = \pi r^2$

$$\frac{dA}{dT} = 2\pi r \frac{dr}{dT}$$

$$\frac{dA}{dT} = 2 \cdot \pi \cdot 4\text{ ft} \cdot 1 \text{ ft/sec} = 8\pi \frac{\text{ft}^2}{\text{sec}}$$

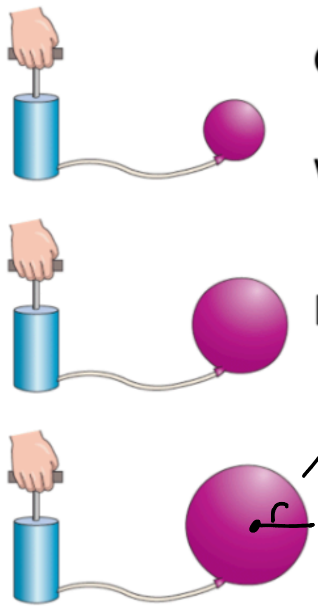
Example 6: Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.

Volume

$$\frac{dV}{dT} = 4.5\ FT^3/min$$

$$\frac{dr}{dT} = \text{Find}$$

when $r = 2$



Given:

Want:

Equation:

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dT} = \frac{4}{\cancel{3}} \cdot \pi \cdot \cancel{3} r^2 \frac{dr}{dT}$$

$$4.5\ FT^3/min = 4\pi r^2 \frac{dr}{dT}$$

$$4.5\ FT^3/min = 4\pi (2FT)^2 \frac{dr}{dT}$$

$$\frac{\frac{9FT^3}{2}/min}{\frac{16\pi FT^2}{1}} = \frac{\cancel{16\pi FT^2} \cdot \frac{dr}{dT}}{\cancel{16\pi FT^2}}$$

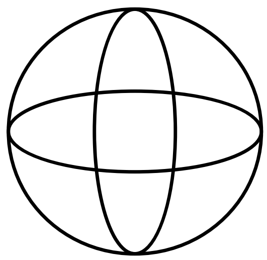
$$\frac{9FT^{\cancel{3}}}{2min} \cdot \frac{1}{16\pi \cancel{FT^2}} = \frac{dr}{dT}$$

$$\frac{9}{32\pi}\ FT/min = \frac{dr}{dT}$$

Your Turn 1: Imagine a spherical balloon is being deflated at a constant rate of 10 cm^3 per second. How fast is the radius changing when the radius is 4 cm?

$\frac{dr}{dt} = ? \rightarrow$ when $r = 4\,cm$

$\frac{dV}{dt} = -10\,cm^3/sec$



$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-10\,cm^3/sec = 4\pi(4\,cm)^2 \frac{dr}{dt}$$

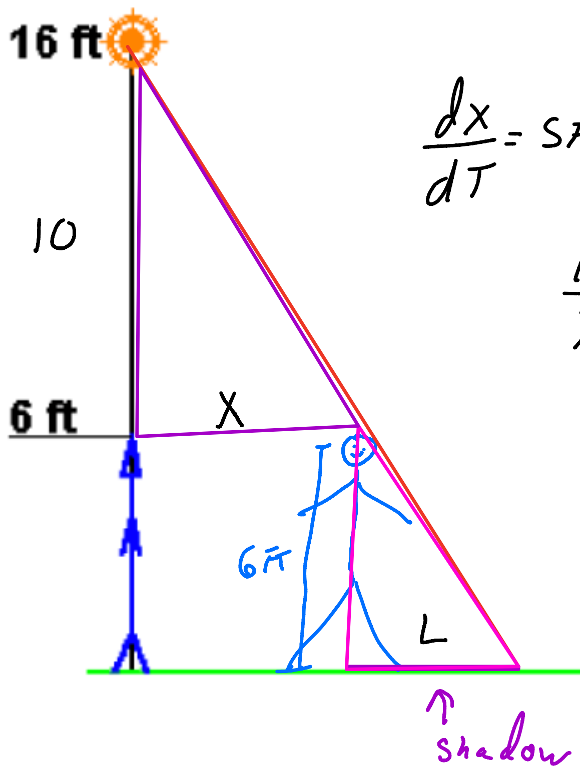
$$\frac{-10\,\cancel{cm^3}/sec}{64\pi\,\cancel{cm^2}} = \frac{\cancel{64\pi\,cm^2}}{\cancel{64\pi\,cm^2}}\frac{dr}{dt}$$

$$\frac{-10}{64\pi}\,cm/sec = \frac{dr}{dt}$$

RELATED RATE DEMO

A man 6 ft tall is directly under a light which is 16 feet high.
He walks to the right at a steady speed of v ft/sec.
As time goes by the length of his shadow increases.

$\frac{dL}{dT}$ = change in shadow Length



$\frac{dx}{dT}$ = speed of walk $= V$

$$\frac{10}{x} = \frac{6}{L}$$

$$6x = 10L$$

$$\frac{6\frac{dx}{dT}}{10\,FT} = \frac{10\frac{dL}{dT}}{10\,FT}$$

$$= \frac{6}{10}\frac{dx}{dT} = \frac{dL}{dT}$$

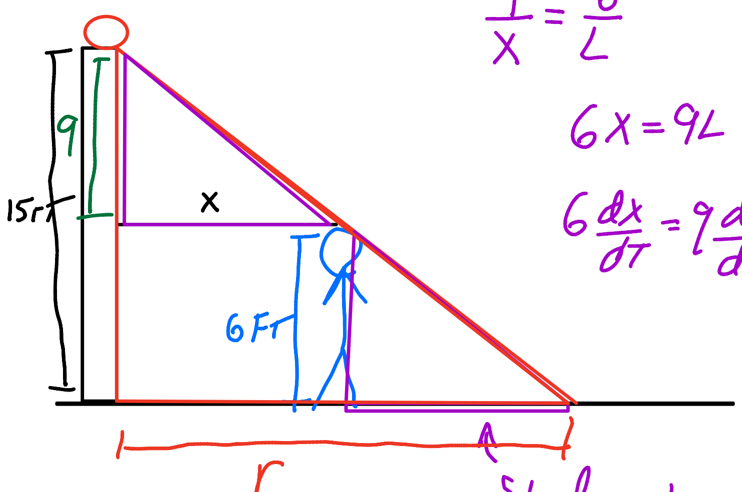
Example 12: A street light is mounted at the top of a 15 ft. pole. A 6 ft. tall man walks away from the pole at a rate of 5 feet per second. $\frac{dx}{dt}$ How fast is the length of his shadow moving when he is 40 feet from the pole? How fast is the tip of his shadow moving?

$$\frac{9}{x} = \frac{6}{L}$$

$$6x = 9L$$

$$6\frac{dx}{dT} = 9\frac{dL}{dT} \Rightarrow \frac{6}{9}\cdot\frac{dx}{dT} = \frac{dL}{dT}$$

$$\frac{dL}{dT} = \frac{6}{9}\cdot 5\,\frac{FT}{sec} = \frac{30}{9} = \frac{10}{3}\,\frac{FT}{sec}$$



Shadow Length = L

$$\frac{15}{r} = \frac{9}{x}$$

$$9r = 15x$$

$$9\frac{dr}{dt} = 15\frac{dx}{dt} \Rightarrow \frac{\cancel{9}\frac{dr}{dT}}{\cancel{9}} = \frac{15\cdot 5\,FT/sec}{9} \Rightarrow \frac{dr}{dT} = \frac{75}{9}\,FT/sec$$

$$\frac{25}{3} = 8\tfrac{1}{3}\,FT/sec$$

$$\frac{dV}{dT} = 5 \text{ in}^3/\text{min}$$

Your Turn 2: A spherical balloon is being inflated at a rate of 5 cubic inches per minute period when the radius of the balloon is 4 inches, how fast is the surface area of the balloon changing?

$$r = 4$$

$$SA = 4\pi r^2 \Rightarrow \frac{dSA}{dT} = 4\cdot\pi\cdot 2r\cdot\frac{dr}{dT} = \underbrace{4\cdot\cancel{\pi}\cdot 2\cdot 4}_{32}\cdot\frac{5}{\cancel{64\pi}_{2}}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dT} = 4\pi r^2\frac{dr}{dT} \Rightarrow 5 = 4\pi(4)^2\cdot\frac{dr}{dT}$$

$$\frac{5}{64\pi} = \frac{dr}{dT}$$

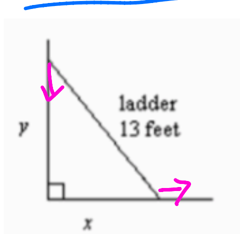
$$\frac{dSA}{dT} = \frac{5}{2} \text{ in}^2/\text{min}$$

Example 13: A 13-ft ladder is leaning against a wall when its base starts to slide away from the wall. By the time its base is 12 ft from the wall, the base is moving at the rate of 5 ft/sec. At what rate is the area of the triangle (formed by the wall, the ladder, and the ground) changing at the same time

$y = 5$

$x = 12$

$dx/dT = 5$



$$x^2 + y^2 = 13^2 = 169$$

$$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 0$$

$$2\cdot 12\cdot 5 + 2\cdot 5\cdot\frac{dy}{dT} = 0$$

$$10\frac{dy}{dT} = -120$$

$$\frac{dy}{dT} = -12 \text{ ft/sec}$$

$$A = \frac{1}{2}\cdot x\cdot y$$

$$\frac{dA}{dT} = \frac{1}{2}\frac{dx}{dT}\cdot y + \frac{1}{2}x\cdot\frac{dy}{dT}$$

$$\frac{dA}{dT} = \frac{1}{2}\cdot 5\cdot 5 + \frac{1}{2}\cdot 12\cdot -12$$

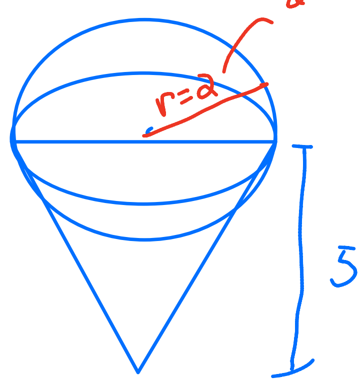
$$\frac{25}{2} - \frac{144}{2}$$

$$\frac{dA}{dT} = \frac{-119}{2} \text{ ft}^2/\text{sec}$$

Your Turn 3: A rainbow snow cone is leaking from its paper cone at a rate of 2 cubic inches per minute. The paper cone's top radius is 3 inches and the paper cone is 5 inches tall. How fast is the radius of the snow cone changing when the radius of the snow cone is 2 inches?

$$\frac{dV}{dT} = -2 \text{ in}^3/\text{min}$$

at Time



$$V = \frac{1}{3} \cdot \pi r^2 \cdot h$$

h is constant

$$V_{\text{half sphere}} = \frac{1}{2} \cdot \frac{4}{3} \pi r^3$$

$$\frac{4}{6} \pi r^3 + \frac{1}{3} \pi r^2 \cdot 5 = V$$